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THE DIFFERENTIAL GAME “COSSACKS–ROBBERS” ON TIME SCALES

In finite-dimensional Euclidean space, we address the problem of simple pursuit of a group of evaders by a group of pursuers in a given time scale with equal opportunities for all participants. The set of controls of each participant is a sphere of unit radius with its center at the origin. The goal of the group of pursuers is to catch all evaders. The goal sets are the origin. The goal of the evaders is the opposite one, namely, for at least one evader to avoid capture. Conditions for solvability of the local and global problems of evasion and the upper and lower estimates of the minimal number of evaders avoiding a given number of pursuers from any initial positions are obtained.

Keywords: differential game, group pursuit, pursuer, evader, evasion problem, time scale.

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Introduction

The work of R. Isaacs [1] laid a foundation for the theory of two-player pursuit–evasion differential games, which has grown to be a profound and insightful theory in which various approaches to analysis of conflict situations [2–8] are proposed. A natural generalization is the situation of conflict interaction of a group of pursuers and a group of evaders, in which the goal of the group of pursuers is to catch a given number of evaders and the goal of the group of evaders is the opposite one.

Reference [9] addressed the problem of simple pursuit of a group of evaders by a group of pursuers with equal opportunities for all participants. The goal of the pursuers was to catch all evaders, and the goal of the evaders was the opposite one. Sufficient conditions for the solvability of local and global evasion problems were obtained and the upper and lower estimates of the minimal number of evaders evading a given number of pursuers from any initial positions were made. An improved variant of the lower estimate was presented in [10]. A generalization of the results of [9] to linear differential games with a constant matrix and a nonstationary matrix was provided in [11] and [12], respectively. In [13] a proof was given of the existence of a price of the game in nonlinear differential games with many participants in a finite time interval and with payoff functions of special form.

Sufficient conditions for the capture of all evaders by a group of pursuers in the nonstationary differential game of simple pursuit in a convex compact set with integral restrictions on the controls of the players were obtained in [14]. In [15], the problem of a group of evaders avoiding a group of pursuers was considered under the assumption that the motion of all participants is simple and that integral restrictions are imposed on the control. It is shown that, if the total energy of the pursuers is smaller than or equal to the total energy of the evaders, an evasion from an encounter occurs.

Reference [16] is concerned with the differential reach–avoid game between two opposing teams in a convex domain consisting of a target domain and a playing zone. The evasion team, which is initially located in the playing zone, strives to send as many team members as possible to the target region, while the pursuit team whose members are initially distributed both over the playing region and over the target region, strives to prevent that by capturing the evaders. The problem under investigation is that of assigning specific pursuers to chase the evaders in such a way that the number of evaders that can be caught before they reach safely the target region is maximized.

Reference [17] treats the problem of pursuit of a group of evaders by a group of pursuers in a probabilistic setting. For each evader, a probability matrix is introduced which estimates the probability of a specific evader being in a particular position. These probabilities are used to construct a forecast of the location of the evaders. The pursuers coordinate their actions and try to decrease the probability matrix.

Reference [18] considers the conflict interaction of a group of pursuers and a group of evaders. It is shown that, if in this conflict the evasion of at least one evader occurs in an infinite time interval, then, with "weak" pursuers being added, the evasion from an encounter will occur in any finite time interval.

Reference [19] is concerned with the dynamical game for two teams: a lady and body-guards against gangsters. The goal of the gangsters is to capture the lady, and the goal of the lady and the body-guards is to prevent that from happening. The body-guards try to intercept the gangsters before they come into immediate proximity to the lady. An approach to solution for a linear system and a quadratic criterion is demonstrated.

Reference [20] addresses the problem of pursuit of several evaders by several pursuers in a convex compact set on a plane. Using a Voronoi partitioning, conditions for the capture of all evaders are obtained.

Reference [21] develops a strategy for cooperation of several unmanned surface ships for pursuing evaders in the presence of a dynamical obstacle ship. To solve the pursuit problem, the pursuers are divided into a pursuit group and an ambush group. The pursuit group drives the evaders into the ambush region and, together with the ambush group, completes encircling the evaders.

Reference [22] is concerned with the differential game of pursuit of a group of evaders by a group of pursuers in three-dimensional space under dynamical perturbations of the environment. It proposes a method for distributing the pursuers into groups each of which catches their own evader. Conditions for solvability of the pursuit problem are given in terms of sets of attainability of the participants.

References [23–25] examine the problem of the capture of a given number of evaders in recurrent differential games, games with fractional derivatives and games on a given time scale under the condition that the evaders use program strategies and that each pursuer catches no more than one evader. Sufficient, and in some cases necessary, conditions for solvability of the pursuit problem are obtained.

Sufficient conditions for the capture of a given number of evaders by a group of pursuers are obtained in [26], implying that each evader is to be caught by a given number of pursuers.

In this paper, we address the problem of conflict interaction of a group of pursuers and a group of evaders in a differential game on a given time scale with simple motion and equal opportunities for all participants. We obtain sufficient conditions for solvability of the local and global evasion problems, and the upper and lower estimates for the minimal number of evaders evading a given number of pursuers from any initial positions.

§ 1. Auxiliary definitions and facts

In this section, we recall some basic facts from the theory of time scales. All results presented below can be found, for example, in [27, 28].

D e f i n i t i o n 1.1. A nonempty closed subset $\mathbb{T} \subset \mathbb{R}^1$ such that $\sup\limits_{t\in\mathbb{T}} t = +\infty$ is called a *time scale*.

D e f i n i t i o n 1.2. Let $\mathbb{T}$ be a time scale. A function $\sigma\colon \mathbb{T} \to \mathbb{R}^1$ of the form

$$\sigma(t) = \inf\{s \in \mathbb{T} \mid s > t\}$$

is called a *translation function*.

D e f i n i t i o n 1.3. A function $f\colon \mathbb{T} \to \mathbb{R}^1$ is said to be Δ-*differentiable* at a point $t \in \mathbb{T}$ if there exists a number $\gamma \in \mathbb{R}^1$ such that for any $\varepsilon > 0$ there exists a neighborhood W of the point t such that the inequality

$$|f(\sigma(t)) - f(s) - \gamma(\sigma(t) - s)| < \varepsilon|\sigma(t) - s|$$

holds for all $s \in \mathbb{T} \cap W$. In this case, the number γ is said to be the Δ-*derivative* of the function f at the point t. The Δ-derivative of the function f at the point t will be denoted as $f^{\Delta}(t) = \gamma$.

D e f i n i t i o n 1.4. A function $f\colon \mathbb{T} \to \mathbb{R}^n$, $f(t) = (f_1(t), \ldots, f_n(t))$ is said to be Δ-*differentiable* at a point $t \in \mathbb{T}$ if all functions $f_1, \ldots, f_n$ are Δ-differentiable at the point t.

Let $\mathbb{T}$ be a time scale, $E \subset \mathbb{T}$. Denote $R(E) = \{t \in E \mid \sigma(t) > t\}$. Then the set $R(E)$ is no more than countable.

D e f i n i t i o n 1.5. A set $E \subset \mathbb{T}$ is said to be Δ-*measurable* if the set

$$\tilde{E} = E \cup \bigcup_{t \in R(E)} (t, \sigma(t))$$

is measurable in the sense of Lebesgue.

D e f i n i t i o n 1.6. A function $f\colon \mathbb{T} \to \mathbb{R}^1$ is said to be Δ-*measurable* on the Δ-measurable set E if the function $\tilde{f}$ of the form

$$\tilde{f}(t) = \begin{cases} f(t), & t \in E, \\ f(t_i), & t \in (t_i, \sigma(t_i)),\ t_i \in R(E), \end{cases}$$

is measurable on the set $\tilde{E}$.

D e f i n i t i o n 1.7. A function $f\colon E \to \mathbb{R}^1$, $E \subset \mathbb{T}$ is called to be Δ-*integrable* on the Δ-measurable set E if the function $\tilde{f}$ is integrable in the sense of Lebesgue on the set $\tilde{E}$. If f is Δ-integrable on the set E, then we define $\int_E f(s)\Delta s$, assuming

$$\int_E f(s)\Delta s = \int_{\tilde{E}} f\,d\mu,$$

where μ is the Lebesgue measure.

§ 2. Formulation of the problem

Suppose we are given a time scale $\mathbb{T}$, $t_0 \in \mathbb{T}$.

In the space $\mathbb{R}^k$ ($k \geqslant 2$) we consider a differential game involving $n + m$ players: n pursuers $P_1, \ldots, P_n$ and m evaders $E_1, \ldots, E_m$. The motion of the players is governed by the laws

$$x_i^{\Delta} = u_i, \quad x_i(t_0) = x_i^0, \quad u_i \in V, \tag{2.1}$$

$$y_j^{\Delta} = v_j, \quad y_j(t_0) = y_j^0, \quad v_j \in V, \tag{2.2}$$

where x_i, y_j, x_i^0, y_j^0, u_i, $v_j \in \mathbb{R}^k$, $i \in I = \{1, \ldots, n\}$, $j \in J = \{1, \ldots, m\}$, $V = \{v \in \mathbb{R}^k\colon \|v\| \leqslant 1\}$. Assume that $x_i^0 \neq y_j^0$ for all $i \in I, j \in J$.

The goal of the group of pursuers is to catch all evaders. The goal of the group of evaders is to prevent this, i. e., to allow at least one of the evaders to evade an encounter.

Let $z^0 = (x_1^0, \ldots, x_n^0, y_1^0, \ldots, y_m^0)$. Denote this game by $\Gamma(n, m, z^0)$. Let σ be a partitioning of the interval $[t_0, +\infty)$ that has no accumulation endpoints. Let $\sigma = \{t_0, t_1, \ldots\}$, with $t_l \in \mathbb{T}$ for all l. Denote $\mathbb{T}_s = [t_s, t_{s+1}) \cap \mathbb{T}$, $\mathbb{T}^s = [t_0, t_s) \cap \mathbb{T}$, where $\{t_s\}$ are the elements of partitioning σ.

D e f i n i t i o n 2.1. A *piecewise–program strategy* V_j of an evader E_j, which corresponds to the partitioning σ, is a family of maps b_j^l, $l = 0, 1, \ldots,$ $j \in J$, that associate to the quantities

$$\Big(t_l, x_i(t_l), i \in I, y_s(t_l), s \in J, \min_{t \in \mathbb{T}^l} \min_i \|x_i(t) - y_s(t)\|\Big) \tag{2.3}$$

a Δ-measurable function $v_j^l(t)$ defined for $t \in \mathbb{T}_l$ and such that $v_j^l(t) \in V$ for all $t \in \mathbb{T}_l$.

D e f i n i t i o n 2.2. A *piecewise–program counterstrategy* U_i of a pursuer P_i, which corresponds to the partitioning σ, is a family of maps c_i^l, $l = 0, 1, \ldots,$ $j \in J$, that associate to the quantities (2.3) and to controls $v_j^l(t)$, $j \in J$, $t \in \mathbb{T}_l$, a Δ-measurable function $u_i^l(t)$ defined for $t \in \mathbb{T}_l$ and such that $u_i^l(t) \in V$ for all $t \in \mathbb{T}_l$.

D e f i n i t i o n 2.3. In the game $\Gamma(n, m, z^0)$ an *evasion* from an encounter occurs if there exist a partitioning σ and piecewise–program strategies $V_1, \ldots, V_m$ of evaders $E_1, \ldots, E_m$ such that for any trajectories $x_1(t), \ldots, x_n(t)$ of pursuers $P_1, \ldots, P_n$ there exists a number $p \in J$ such that

$$y_p(t) \neq x_i(t) \quad \forall i \in I,\ t \in \mathbb{T},$$

where $y_p(t)$ is the trajectory of the evader E_p that takes place in this situation.

D e f i n i t i o n 2.4. A *capture* occurs in the game $\Gamma(n, m, z^0)$ if there exists $T > t_0$, $T \in \mathbb{T}$ such that for any partitioning σ and any strategies $V_1, \ldots, V_m$ of evaders $E_1, \ldots, E_m$ there exist piecewise–program counterstrategies $U_1, \ldots, U_n$ of pursuers $P_1, \ldots, P_n$ corresponding to the partitioning σ such that there exist time instants $\tau_1, \ldots, \tau_m \in [t_0, T) \cap \mathbb{T}$ and numbers $s_1, \ldots, s_m \in I$ for which the following equations hold:

$$y_j(\tau_j) = x_{s_j}(\tau_j), \quad j \in J,$$

where $x_{s_j}(t), s_j \in I$, $y_j(t), j \in J$, are the trajectories of players $P_{s_j}, s_j \in I$, $E_j, j \in J$, that take place in this situation.

§ 3. The local evasion problem

In this section, we present some sufficient conditions for an evasion from an encounter in the game $\Gamma(n, m, z^0)$.

For a hyperplane H, we denote by H^+ and H^- closed half-spaces defined by this hyperplane.

L e m m a 3.1. *Suppose there exists a hyperplane H such that*
a) $x_i^0 \in H^-$ *for all* $i \in I$;
b) $y_1^0 \in H^+$.
Then an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$.

P r o o f. Let q be the unit vector of the normal of the hyperplane H which is directed to H^+, and let $u_i(t)$, $i \in I$, be arbitrary controls of the pursuers. Define the control of the evader E_1, assuming $v_1(t) = q$ for all $t \in \mathbb{T}$. Define the controls of the other evaders arbitrarily. From the systems (2.1) and (2.2), we obtain

$$y_1(t) = y_1^0 + K(t)q, \quad x_i(t) = x_i^0 + \int_{t_0}^{t} u_i(s)\Delta s, \quad \text{where } K(t) = \int_{t_0}^{t} \Delta s.$$

Define the functions

$$\hat{u}_i(t) = \frac{1}{K(t)} \int_{t_0}^{t} u_i(s)\Delta s.$$

Then $\|\hat{u}_i(t)\| \leqslant 1$ for all $t \in \mathbb{T}$. Next, we have ($z_i^0 = x_i^0 - y_1^0$),

$$\|x_i(t) - y_1(t)\| = \|z_i^0 - K(t)q + \hat{u}_i(t)K(t)\| \geqslant \|z_i^0 - K(t)q\| - K(t) =$$
$$= \sqrt{\|z_i^0\|^2 - 2K(t)(z_i^0, q) + K^2(t)} - K(t) > 0,$$

since $(z_i^0, q) \leqslant 0$ for all $i \in I$. This proves the lemma. □

C o r o l l a r y 3.1. *Let $H(t) = H + (t - t_0)q$. Then for any controls $u_i(t)$, $i \in I$, of pursuers P_i, $i \in I$, and for all $t \in \mathbb{T}$ the inclusion $x_i(t) \in H^-(t)$ holds, where $H^-(t)$ is a closed half-space defined by the hyperplane $H(t)$ such that $H^- \subset H^-(t)$.*

P r o o f. Indeed, for all $t \in \mathbb{T}$ and all $i \in I$ the following inequality holds:

$$(x_i(t) - y(t), q) = (z_i^0, q) + K(t)\big[(\hat{u}_i(t), q) - 1\big] \leqslant (z_i^0, q) \leqslant 0.$$

We note that $y(t) \in H(t)$ for all $t \in \mathbb{T}$. This proves the corollary. □

L e m m a 3.2. *Suppose there exist hyperplanes H_1, H_2 and a number $l \in I$ such that*
a) $H_1 || H_2$, $H_2^+ \subset H_1^+$;
b) $x_l^0 \in H_2^+$, $x_i^0 \in H_1^-$ *for all* $i \in I$, $i \neq l$;
c) $y_1^0 \in H_1$, $y_2^0 \in H_2$;
d) *the projections of all points x_i^0, $i \in I$, y_1^0, y_2^0 onto the hyperplane H_1 are pairwise different. Then an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$.*

P r o o f. Let q be the unit vector of the normal of the hyperplane H_1 directed to H_1^+. Suppose that $w_1(t) = y_1^0 + (t - t_0)q$, $w_2(t) = y_2^0 - (t - t_0)q$, $d_1(t)$ is the distance from the point $w_1(t)$ to the hyperplane H_1, $d_2(t)$ is the distance from the point $w_2(t)$ to the hyperplane H_1. We note that $d_1(t_0) = 0$, $d_2(t_0) > 0$, d_1 is an increasing function, and d_2 is a decreasing function.

Two cases are possible.

1. There exists $\tau \in \mathbb{T}$ for which $d_1(\tau) = d_2(\tau)$. Then we define the controls of evaders E_1 and E_2 as follows. Assume that $v_1(t) = q$ for all $t \in \mathbb{T}$.

$$v_2(t) = \begin{cases} -q, & t \in [t_0, \tau] \cap \mathbb{T}, \\ q, & t \in (\tau, +\infty) \cap \mathbb{T}. \end{cases}$$

Let us define the controls of the other evaders arbitrarily. We prove that an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$. It follows from Lemma 3.1 that $x_i(t) \neq y_1(t)$ for all $i \neq l$, $t \in \mathbb{T}$. In addition, by virtue of Lemma 3.1 and Corollary 3.1, $x_l(t) \neq y_1(t)$, $x_l(t) \neq y_2(t)$ for all $t \in \mathbb{T}^{\tau}$. It follows from Corollary 3.1 that $x_i(t) \neq y_2(t)$ for all $t \in \mathbb{T}$, $i \neq l$. We show that on the set $(\tau, +\infty) \cap \mathbb{T}$ the pursuer P_l catches no more than one of the evaders E_1 and E_2. For all $t \geqslant \tau$ we have $y_1(t), y_2(t) \in H_1(t) = H + (t - t_0)q$. Therefore, if a capture of $E_1(E_2)$ by the pursuer P_l occurs at the time instant τ_1, then, at the time instant τ_1, the conditions of Lemma 3.1

will be satisfied for the evader $E_2(E_1)$. Thus, we have proved that an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$.

2. For all $t \in \mathbb{T}$ the following inequality holds: $d_1(t) \neq d_2(t)$. Then there exist $\tau_1, \tau_2 \in \mathbb{T}$, $\tau_1 < \tau_2$ for which $(\tau_1, \tau_2) \cap \mathbb{T} = \emptyset$, $d_1(\tau_1) < d_2(\tau_1)$, $d_1(\tau_2) > d_2(\tau_2)$. Let us define the controls of the evaders E_1 and E_2 as follows. Assume that $v_1(t) = q$ for all $t \in \mathbb{T}$.

If $d_1(\tau_2) > d_2(\tau_1)$, then we assume

$$v_2(t) = \begin{cases} -q, & t \in [t_0, \tau_1) \cap \mathbb{T}, \\ \dfrac{q\big(d_1(\tau_2) - d_2(\tau_1)\big)}{\tau_2 - \tau_1}, & t = \tau_1, \\ q, & t \in [\tau_2, +\infty) \cap \mathbb{T}. \end{cases}$$

If $d_2(\tau_1) > d_1(\tau_2)$, then we assume

$$v_2(t) = \begin{cases} -q, & t \in [t_0, \tau_1) \cap \mathbb{T}, \\ \dfrac{-q\big(d_2(\tau_1) - d_1(\tau_2)\big)}{\tau_2 - \tau_1}, & t = \tau_1, \\ q, & t \in [\tau_2, +\infty) \cap \mathbb{T}. \end{cases}$$

Let us define the controls of the other evaders arbitrarily. We note that, if $d_1(\tau_2) > d_2(\tau_1)$, then

$$0 \leqslant \frac{d_1(\tau_2) - d_2(\tau_1)}{\tau_2 - \tau_1} \leqslant \frac{d_1(\tau_2) - d_1(\tau_1)}{\tau_2 - \tau_1} = 1.$$

Similarly, if $d_2(\tau_1) > d_1(\tau_2)$, then

$$0 \leqslant \frac{d_2(\tau_1) - d_1(\tau_2)}{\tau_2 - \tau_1} \leqslant \frac{d_2(\tau_1) - d_2(\tau_2)}{\tau_2 - \tau_1} = 1.$$

Therefore, the function $v_2(\cdot)$ satisfies the condition $\|v_2(t)\| \leqslant 1$ for all $t \in \mathbb{T}$. We prove that in the game $\Gamma(n, m, z^0)$ an evasion occurs in this case as well. By virtue of Corollary 3.1, pursuers P_i, $i \in I$, $i \neq l$, catch none of the evaders E_1 and E_2. On the set $\mathbb{T}^{\tau_1}$, the pursuer P_l catches none of the evaders E_1 and E_2. If the pursuer P_l performs a capture of one of the evaders E_1 and E_2 at the time instant $\tau \geqslant \tau_1$, then at the time instant τ the pursuer P_l will be on the hyperplane $H(\tau)$ and hence, by virtue of Lemma 3.1, the pursuer P_l will not be able to perform a capture of the second evader. Thus, an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$. This proves the lemma.

C o r o l l a r y 3.2. *Suppose that in the game $\Gamma(n, m, z^0)$ there exist hyperplanes H_1, H_2 and sets $I_0 \subset I$, $J_0 \subset J$ such that*

a) $H_1 || H_2$, $H_2^+ \subset H_1^+$;

b) $|J_0| \geqslant |I_0| + 1$, *where $|J|$ denotes the number of elements of the set J*;

c) $x_i^0 \in H_2^+$, $i \in I_0$, $x_i^0 \in H_1^-$, $i \notin I_0$;

d) $y_j^0 \in H_1^+ \cap H_2^-$, $j \in J_0$;

e) *the projections of all points x_i^0, $i \in I$, y_j^0, $j \in J_0$, onto the hyperplane H_1 are pairwise different.*

Then an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$.

P r o o f. Assume that $J_0 = \{1, \dots, l\}$. Let q be the unit vector of the normal of the hyperplane H_1 which is directed to H_1^+. Let y_1^0 be one of the points $y_j^0, j \in J_0$, that is nearest to H_1. Suppose that $w_1(t) = y_1^0 + (t - t_0)q$, $w_j(t) = y_j^0 - (t - t_0)q$, $j \in J_0$, $j \neq 1$, and let $d_j(t)$ be the distance from the point $w_j(t)$ to the hyperplane H_1. Define the control v_1 of the evader E_1, assuming $v_1(t) = q$ for all $t \in \mathbb{T}$. Define the controls of evaders E_j, $j \in J_0$, $j \neq 1$, as follows.

1. If there exists a time instant $\tau_j \in \mathbb{T}$ for which $d_1(\tau_j) = d_1(\tau_j)$, then we assume

$$v_j(t) = \begin{cases} -q, & t \in [t_0, \tau_j] \cap \mathbb{T}, \\ q, & t \in (\tau_j, +\infty) \cap \mathbb{T}. \end{cases}$$

2. Suppose that the inequality $d_j(t) \neq d_1(t)$ holds for all $t \in \mathbb{T}$. Then there exist $\tau_{1j}, \tau_{2j} \in \mathbb{T}$, $\tau_{1j} < \tau_{2j}$, for which $(\tau_{1j}, \tau_{2j}) \cap \mathbb{T} = \emptyset$, $d_1(\tau_{1j}) < d_j(\tau_{1j})$, $d_1(\tau_{2j}) > d_j(\tau_{2j})$.

Then, if $d_1(\tau_{2j}) > d_j(\tau_{1j})$, we assume

$$v_j(t) = \begin{cases} -q, & t \in [t_0, \tau_{1j}) \cap \mathbb{T}, \\ \dfrac{q\big(d_1(\tau_{2j}) - d_j(\tau_{1j})\big)}{\tau_{2j} - \tau_{1j}}, & t = \tau_{1j}, \\ q, & t \in [\tau_{2j}, +\infty) \cap \mathbb{T}. \end{cases}$$

If $d_j(\tau_{1j}) > d_1(\tau_{2j})$, then we assume

$$v_j(t) = \begin{cases} -q, & t \in [t_0, \tau_{1j}) \cap \mathbb{T}, \\ \dfrac{-q\big(d_j(\tau_{1j}) - d_1(\tau_{2j})\big)}{\tau_{2j} - \tau_{1j}}, & t = \tau_{1j}, \\ q, & t \in [\tau_{2j}, +\infty) \cap \mathbb{T}. \end{cases}$$

Define the controls of the other evaders, E_j and $j \notin J_0$, in an arbitrary way. By virtue of Lemma 3.2, the function $v_j(\cdot)$ satisfies the condition $\|v_j(t)\| \leqslant 1$ for all $t \in \mathbb{T}$. By virtue of Lemma 3.2, pursuers P_j, $j \notin I_0$, catch none of the evaders E_j, $j \in J_0$. Each of the pursuers P_j, $j \in I_0$, can catch no more than one of the evaders E_j, $j \in J_0$. Therefore, by virtue of condition b) of the corollary, at least one of the evaders E_j, $j \in J_0$, will avoid a capture. Consequently, an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$. This proves the corollary.

R e m a r k 3.1. The strategies of the evaders constructed in proving Corollary 3.2, which guarantee an evasion from an encounter, possess the following property. Let H be a hyperplane that is parallel to the hyperplane H_1 and passes through point y_1^0, where y_1^0 is one of the points y_j^0, $j \in J_0$, that is nearest to H_1, and let $H(t) = H + (t - t_0)q$. Then the evader E_1 is on the hyperplane $H(t)$ at each time instant $t \in \mathbb{T}$. The motion of each of the other evaders E_j, $j \in J_0$, $j \neq 1$, consists of two stages. In the first stage, each of evaders E_j, $j \neq 1$, moves along the normal $-q$ to the hyperplane H so as to be on the hyperplane $H(\tau)$ at some time instant τ. In the second stage, evader E_j, $j \in J_0$, $j \neq 1$, moves along the normal q to the hyperplane H and is thus located on the hyperplane $H(t)$ for all $t > \tau$.

§ 4. The global evasion problem

T h e o r e m 4.1. *For any natural number p and any natural number $m \geqslant p \cdot 2^p + 2$, an evasion from an encounter occurs in the game $\Gamma(2^p + 1, m, z^0)$.*

P r o o f. Let $n = 2^p + 1$, $I = \{1, \ldots, n\}$, $J = \{1, \ldots, m\}$, $x_1^0, \ldots, x_n^0$ be the initial positions of the pursuers and let $y_1^0, \ldots, y_m^0$ be the initial positions of the evaders. Assume that all points $x_1^0, \ldots, x_n^0, y_1^0, \ldots, y_m^0$ are pairwise different. Let q be a unit vector such that $(q, x_\alpha^0 - x_\beta^0) \neq 0$ for all $\alpha \neq \beta$, $(q, x_i^0 - y_j^0) \neq 0$ for all i, j, and $(q, y_r^0 - y_s^0) \neq 0$ for all $r \neq s$. Suppose that $H_1, \ldots, H_n$ are hyperplanes with the normal q for which $x_i^0 \in H_i$ for all $i \in I$, and H_i^-, H_i^+ are closed half-spaces defined by the hyperplane $H_i, i \in I$, with $H_i^+ \subset H_{i+1}^+$ for all $i = 1, \ldots, n - 1$. Assume that q is directed to H_n^+.

If at least one of the points y_j^0 belongs to $H_1^+ \cup H_n^-$, then, by virtue of Lemma 3.1, an evasion from an encounter occurs in the game $\Gamma(n, m, z^0)$. Further, let $y_j^0 \in H_1^- \cap H_n^+$ for all j. The theorem will be proved by the method of mathematical induction with respect to p.

1. $p = 1$. Define the sets $J_1 = \{j \mid y_j^0 \in H_2^- \cap H_3^+\}$, $J_2 = \{j \mid y_j^0 \in H_1^- \cap H_2^+\}$. Construct auxiliary controls $\overline{v}_j(t)$ for evaders E_j, $j \in J$. For the group of evaders E_j, $j \in J_1$, $\overline{v}_j(t)$ are controls constructed in accordance with Corollary 3.2 with respect to the hyperplane H_3 and the normal vector q directed to H_3^+. For the group of evaders E_j, $j \in J_2$, $\overline{v}_j(t)$ are controls constructed in accordance with Corollary 3.2 with respect to the hyperplane H_1 and the normal vector $-q$ directed to H_1^-.

Suppose that y_α^0 is one of the points y_j^0, $j \in J_1$, that is nearest to the hyperplane H_3, and H_α is a hyperplane that is parallel to H_3 and passes through y_α^0. Next, suppose that y_β^0 is one of the points y_j^0, $j \in J_2$, that is nearest to the hyperplane H_1; let H_β be a hyperplane that is parallel to H_1 and passes through y_β^0; let $H_\alpha(t) = H_\alpha + (t - t_0)q$, $H_\beta(t) = H_\beta - (t - t_0)q$; let τ_1 be the first time instant when all evaders E_j, $j \in J_1$, reach the hyperplane $H_\alpha(\tau_1)$ when using controls $\overline{v}_j(t)$, $j \in J_1$; let τ_2 be the first time instant when all evaders E_j, $j \in J_2$, reach the hyperplane $H_\beta(\tau_2)$ when using controls $\overline{v}_j(t)$, $j \in J_2$; let $\tau = \max\{\tau_1, \tau_2\}$; $d_\alpha(t)$ be the distance from the point $w_\alpha(t) = y_\alpha(\tau) + (t - \tau)q$ to the hyperplane $H_\alpha(\tau)$; let $d_\beta(t)$ be the distance from the point $w_\beta(t) = y_\beta(\tau) - (t - \tau)q$ to the hyperplane $H_\alpha(\tau)$. Note that $\tau \in \mathbb{T}$, $d_\alpha(\tau) = 0$, $d_\beta(\tau) > 0$. Two cases are possible.

1.1. There exists a time instant $\tau_0 \in \mathbb{T}$ for which $d_\alpha(\tau_0) = d_\beta(\tau_0)$. We define the controls of evaders E_j, $j \in J$, as follows: $v_\alpha(t) = q$ for all $t \in \mathbb{T}$, and $v_j(t) = \overline{v}_j(t)$ for all $t \in \mathbb{T}$, $j \in J_1$, $j \neq \alpha$.

For $j \in J_2$, we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \tau_0] \cap \mathbb{T}, \\ q, & t \in (\tau_0, +\infty) \cap \mathbb{T}. \end{cases}$$

1.2. For all $t \in \mathbb{T}$, $t > \tau$, the inequality $d_\alpha(t) \neq d_\beta(t)$ holds. Then there exist time instants τ^1, $\tau^2 \in \mathbb{T}$, $\tau < \tau^1 < \tau^2$, such that $d_\alpha(\tau^1) < d_\beta(\tau^1)$, $d_\alpha(\tau^2) > d_\beta(\tau^2)$. Define the controls of evaders E_j, $j \in J$, as follows.

If $j \in J_1$, then we assume $v_j(t) = \overline{v}_j(t)$ for all $t \in \mathbb{T}$. If $d_\alpha(\tau^2) \geqslant d_\beta(\tau^1)$ and $j \in J_2$, then we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \tau^1) \cap \mathbb{T}, \\ \dfrac{q(d_\alpha(\tau^2) - d_\beta(\tau^1))}{\tau^2 - \tau^1}, & t = \tau^1, \\ q, & t \in [\tau^2, +\infty) \cap \mathbb{T}. \end{cases}$$

If $d_\beta(\tau^1) > d_\alpha(\tau^2)$ and $j \in J_2$, then we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \tau^1) \cap \mathbb{T}, \\ \dfrac{-q(d_\beta(\tau^1) - d_\alpha(\tau^2))}{\tau^2 - \tau^1}, & t = \tau^1, \\ q, & t \in [\tau^2, +\infty) \cap \mathbb{T}. \end{cases}$$

2. Now assume that the strategies $V_1, \ldots, V_m$ of evaders $E_1, \ldots, E_m$ are constructed for all $p < r$.

3. Construct the strategies $V_1, \ldots, V_m$ of evaders $E_1, \ldots, E_m$ for $p = r$. Let $n_1 = 2^{r-1} + 1$. $J_1 = \{j \mid y_j^0 \in H_{n_1}^- \cap H_n^+\}$, $J_2 = \{j \mid y_j^0 \in H_1^- \cap H_{n_1}^+\}$. By virtue of the induction assumption, in the game $\Gamma(n_1, |J_1|, \overline{z}^0)$, where $\overline{z}^0 = (\overline{x}^0, \overline{y}^0)$, $\overline{x}^0 = (x_{n_1}^0, \ldots, x_n^0)$, $\overline{y}^0 = (y_j^0, j \in J_1)$, the strategies $\overline{V}_j$, $j \in J_1$, of evaders E_j, $j \in J_1$, are defined. Similarly, in the game $\Gamma(n_1, |J_2|, \hat{z}^0)$, where $\hat{z}^0 = (\hat{x}^0, \hat{y}^0)$, $\hat{x}^0 = (x_1^0, \dots, x_{n_1}^0)$, $\hat{y}^0 = (y_j^0, j \in J_2)$, the strategies $\overline{V}_j$, $j \in J_2$ of evaders E_j, $j \in J_2$, are defined. Next, suppose that y_α^0 is one of the points y_j^0, $j \in J_1$, that is nearest to the hyperplane H_n, and H_α is a hyperplane that is parallel to H_n and passes through y_α^0, y_β^0 is one of the points $y_j^0, j \in J_2$, that is nearest to the hyperplane H_1. Also, suppose that H_β is a hyperplane that is parallel to H_1 and passes through y_β^0; $H_\alpha(t) = H_\alpha + (t - t_0)q$; $H_\beta(t) = H_\beta - -(t-t_0)q$; $t_1 \in \mathbb{T}$ is the first time instant when all evaders E_j, $j \in J_1$, reach the hyperplane $H_\alpha(t_1)$ when using strategies $\overline{V}_j(t)$, $j \in J_1$; $t_2 \in \mathbb{T}$ is the first time instant when all evaders E_j, $j \in J_2$, reach the hyperplane $H_\beta(t_2)$ when using strategies $\overline{V}_j(t)$, $j \in J_2$; $\overline{t} = \max\{t_1, t_2\}$; $d_\alpha(t)$ is the distance from the point $w_\alpha(t) = y_\alpha(\overline{t}) + (t - \overline{t})q$ to the hyperplane $H_\alpha(\overline{t})$; $d_\beta(t)$ is the distance from the point $w_\beta(t) = y_\beta(\overline{t}) - (t - \overline{t})q$ to the hyperplane $H_\alpha(\overline{t})$. Note that $\overline{t} \in \mathbb{T}$, $d_\alpha(\overline{t}) = 0$, $d_\beta(\overline{t}) > 0$. Two cases are possible.

3.1. There exists a time instant $\hat{t} \in \mathbb{T}$, $\hat{t} > \overline{t}$, for which $d_\alpha(\hat{t}) = d_\beta(\hat{t})$. Then we define the strategies $V_1, \dots, V_m$ of evaders $E_1, \dots, E_m$ in the game $\Gamma(n, m, z^0)$ as follows. For all $j \in J_1$, we assume $v_j(t) = \overline{v}_j(t)$, $t \in \mathbb{T}$. If $j \in J_2$, then we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \hat{t}] \cap \mathbb{T}, \\ q, & t \in (\hat{t}, +\infty) \cap \mathbb{T}. \end{cases}$$

3.2. For all $t \in \mathbb{T}$, $t > \hat{t}$, the inequality $d_\alpha(t) \neq d_\beta(t)$ holds. Then there exist time instants $\hat{\tau}_1$, $\hat{\tau}_2 \in \mathbb{T}$, $\hat{t} < \hat{\tau}_1 < \hat{\tau}_2$, $(\hat{\tau}_1, \hat{\tau}_2) \cap \mathbb{T} = \emptyset$, for which $d_\alpha(\hat{\tau}_1) < d_\beta(\hat{\tau}_2)$, $d_\alpha(\hat{\tau}_2) > d_\beta(\hat{\tau}_2)$. Define the controls of evaders E_j, $j \in J$, as follows. If $j \in J_1$, then we assume $v_j(t) = \overline{v}_j(t)$ for all $t \in \mathbb{T}$. If $d_\alpha(\hat{\tau}_2) > d_\beta(\hat{\tau}_1)$ and $j \in J_2$, then we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \hat{\tau}_1) \cap \mathbb{T}, \\ \dfrac{q(d_\alpha(\hat{\tau}_2) - d_\beta(\hat{\tau}_1))}{\hat{\tau}_2 - \hat{\tau}_1}, & t = \hat{\tau}_1, \\ q, & t \in [\hat{\tau}_2, +\infty) \cap \mathbb{T}. \end{cases}$$

If $d_\beta(\hat{\tau}_1) > d_\alpha(\hat{\tau}_2)$ and $j \in J_2$, then we assume

$$v_j(t) = \begin{cases} \overline{v}_j(t), & t \in [t_0, \hat{\tau}_1) \cap \mathbb{T}, \\ \dfrac{-q(d_\beta(\hat{\tau}_1) - d_\alpha(\hat{\tau}_2))}{\hat{\tau}_2 - \hat{\tau}_1}, & t = \hat{\tau}_1, \\ q, & t \in [\hat{\tau}_2, +\infty) \cap \mathbb{T}. \end{cases}$$

Let $\hat{\tau}_0 \in \mathbb{T}$ be the time instant when all evaders E_j, $j \in J$, reach the hyperplane $H_\alpha(\hat{\tau}_0)$. Using the induction and Lemma 3.2, we find that on $[t_0, \hat{\tau}_0] \cap T$ pursuers $P_1, \dots, P_n$ catch no more than $(p-1)2^p + 1$ evaders. By virtue of Lemma 3.2, on $[\hat{\tau}_0, +\infty) \cap T$ pursuers $P_1, \dots, P_n$ catch no more than 2^p evaders. Therefore, the total number of evaders that pursuers $P_1, \dots, P_n$ catch is no more than $p \cdot 2^p + 1$. This proves the theorem. □

R e m a r k 4.1. In accordance with the constructed strategies, the motion of the evaders occurs as follows. Originally the phase space is divided into layers by parallel hyperplanes in such a way that in each hyperplane there is one pursuer and inside the layer formed by two neighboring hyperplanes there are no pursuers. In the first step, the evaders that lie in the layer formed by two neighboring hyperplanes move so that all of them are on a hyperplane parallel to these hyperplanes. In the second step, the evaders situated on two neighboring hyperplanes come together, and so on. Before the last step, all evaders lie in two parallel hyperplanes. In the last step, the evaders come together so as to be in one hyperplane, and they keep moving along the normal to this hyperplane with the maximum velocity.

Define the function $f\colon N \to N$

$$f(n) = \min\{m \mid \text{in the game } \Gamma(n, m, z^0) \text{ an evasion from an encounter occurs from any initial positions } z^0\}.$$

Theorem 4.2. *There exists a constant* $C_1 > 0$ *such that for any natural* n, $n \neq 1$, *the following inequality holds:*

$$f(n) \leqslant C_1 n \ln n.$$

Proof. It follows from Theorem 4.1 that for any natural p the following inequality holds:

$$f(2^p + 1) \leqslant p \cdot 2^p + 2.$$

Let $n \in N$, $n \neq 1$. Take a natural number p such that

$$2^{p-1} < n < 2^p + 1.$$

Then

$$f(n) \leqslant f(2^p + 1) \leqslant p \cdot 2^p + 2 \leqslant C_1 n \ln n,$$

where $C_1 = \frac{5}{\ln 2}$. This proves the theorem. □

Let $\operatorname{Int} A$ and $\operatorname{co} A$ denote the interior and the convex hull of the set A, respectively.

Theorem 4.3 (see [29, p. 7]). *Suppose that in the game* $\Gamma(n, 1, z^0)$

$$y_1^0 \in \operatorname{Int}\operatorname{co}\{x_1^0, \ldots, x_n^0\}.$$

Then a capture occurs in the game $\Gamma(n, 1, z^0)$.

Theorem 4.4. *There exists a constant* $C_2 > 0$ *such that for any* $n \in N$ *the following inequality holds:*

$$f(n) \geqslant C_2 n \ln n.$$

This theorem is proved along the same lines as the theorem of [9] using Theorem 4.3.

Corollary 4.1. *For any natural number* l *there exist natural numbers* n, m *and a vector of initial positions,* z^0, *such that* $m - n > l$ *and a capture occurs in the game* $\Gamma(n, m, z^0)$.

Corollary 4.2. *For any natural number* l *there exist natural numbers* n, m *such that in the game* $\Gamma(n, m, z^0)$ *an evasion from an encounter occurs for any* z^0, *and in the game* $\Gamma(n + 1, m + l, z^1)$ *a capture occurs for some* z^1.

The proofs of the last two corollaries are identical to those of the corresponding corollaries in [9].

Remark 4.2. We note that the results of [9] are a consequence of the results of this paper for $\mathbb{T} = \mathbb{R}^1$.

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Е. С. Можегова, Н. Н. Петров
Дифференциальная игра «казаки–разбойники» во временных шкалах

Ключевые слова: дифференциальная игра, групповое преследование, преследователь, убегающий, задача уклонения, временная шкала.

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В конечномерном евклидовом пространстве рассматривается задача простого преследования группой преследователей группы убегающих в заданной временной шкале с равными возможностями всех участников. Множество управлений каждого участника — шар радиуса единица с центром в начале координат. Целью группы преследователей является поимка всех убегающих. Целевые множества — начало координат. Цель группы убегающих противоположна, то есть предоставить возможность хотя бы одному из убегающих избежать поимки. Получены условия разрешимости локальной и глобальной задач уклонения, а также оценки сверху и снизу для наименьшего числа убегающих, уклоняющихся от заданного числа преследователей из любых начальных позиций.

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